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MATHEMATICS

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Department of School Education

Untouchability is Inhuman and a Crime

Chapter

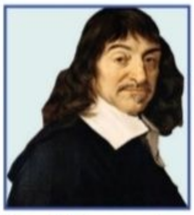
5

Two Dimensional Analytical Geometry-II

"Divide each difficulty into as many parts as is feasible and necessary to resolve it"

René Descartes

5.1 Introduction



René Descartes
1596 – 1650

Analytical Geometry of two dimension is used to describe geometric objects such as **point, line, circle, parabola, ellipse,** and **hyperbola** using **Cartesian coordinate system**.Two thousand years ago (≈ 2– 1 BC (BCE)), the ancient Greeks studied **conic** curves, because studying them elicited ideas that were exciting, challenging, and interesting. They could not have imagined the applications of these curves in the later centuries.

Solving problems by the method of Analytical Geometry was systematically developed in the first half of the 17th century majorly, by Descartes and also by other great mathematicians like Fermat, Kepler, Newton, Euler, Leibniz, l'Hôpital, Clairaut, Cramer, and the Jacobis.

Analytic Geometry grew out of need for establishing **algebraic techniques** for solving **geometrical problems** and the development in this area has conquered industry, medicine, and scientific research.

The theory of Planetary motions developed by Johannes Kepler, the German mathematician cum physicist stating that all the planets in the solar system including the earth are moving in elliptical orbits with Sun at one of a foci, governed by inverse square law paved way to established work in Euclidean geometry. Euler applied the co-ordinate method in a systematic study of space curves and surfaces, which was further developed by Albert Einstein in his theory of relativity.

Applications in various fields encompassing **gears**, **vents** in dams, **wheels** and circular geometry leading to trigonometry as application based on properties of circles; arches, dish, **solar cookers**, **head-lights**, **suspension bridges,** and **search lights** as application based on properties of parabola; arches, **Lithotripsy** in the field of Medicine, **whispering galleries,** Ne-de-yag lasers and gears as application based on properties of ellipse; and telescopes, cooling towers, **spotting locations** of ships or aircrafts as application based on properties of hyperbola, to name a few.

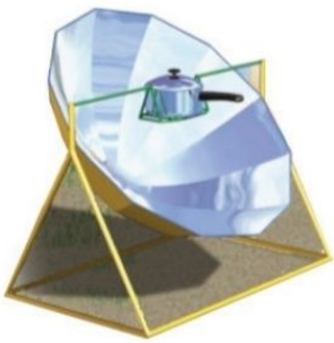


Fig. 5.1

Fig. 5.2

Fig. 5.3

Conics

Defn of Conics:

A Conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed line not containing the fixed point.

The fixed point is called focus, the fixed line is called directrix and the constant ratio is called eccentricity, which is denoted by e.

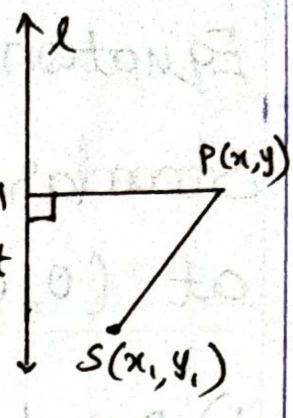
i) If this constant $e=1$ then the conic is called a Parabola.

ii) If this constant $e<1$ then the Conic is called a ellipse

iii) If this constant $e>1$ then the Conic is called a Hyperbola.

The general equation of a Conic:

Let $S(x_1, y_1)$ be the focus, l the directrix, and e be the eccentricity.



Let $P(x,y)$ be the moving point

$$\frac{SP}{PM} = \text{constant} = e.$$

$$SP = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$ and

PM = perpendicular distance from $P(x,y)$ to the line $lx+my+n=0$

$$= \left|\frac{lx+my+n}{\sqrt{l^2+m^2}}\right|$$

The general second-degree equation is $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$.

$$B^2-4AC = 4(e^2-1)$$

(i) $B^2-4AC=0 \Leftrightarrow e=1 \Leftrightarrow$ the conic is a parabola.

ii) $B^2-4AC<0 \Leftrightarrow 0<e<1 \Leftrightarrow$ the conic is an ellipse.

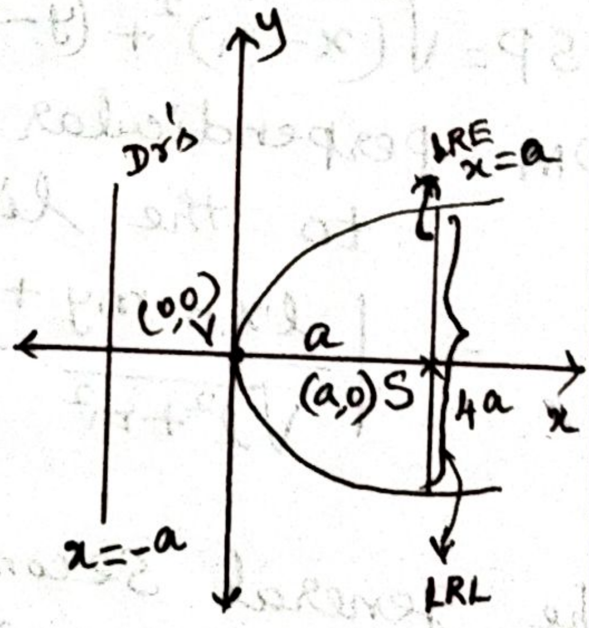
iii) $B^2-4AC>0 \Leftrightarrow e>1 \Leftrightarrow$ the conic is a hyperbola.

Parabola: $(e=1)$

Equation of a parabola in Standard form with vertex at $(0,0)$:

1) parabola open the rightward:

$$y^2 = 4ax$$



i) Axis : $y=0$

ii) Vertex: $V(0,0)$

iii) Focus: $S(a,0)$

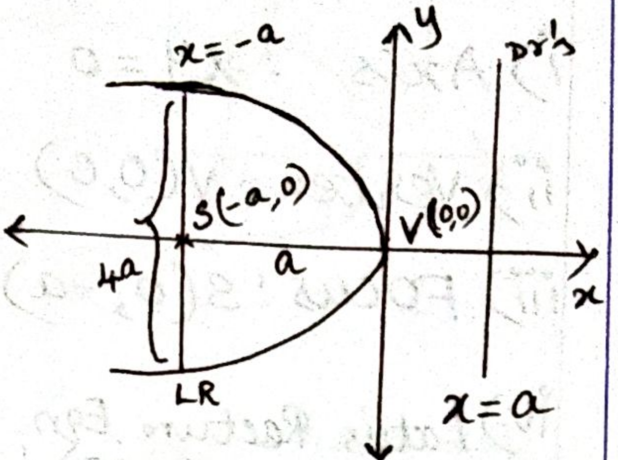
iv) Latus Rectum Eqn (LRE) : $x=a$

v) Directrix (Dr's) : $x=-a$

vi) Latus Rectum Length (LRL) : $4a$

2) Parabola open the leftward:

$$y^2 = -4ax$$



i) Axis : $y=0$

ii) vertex : $V(0,0)$

iii) Focus : $S(-a,0)$

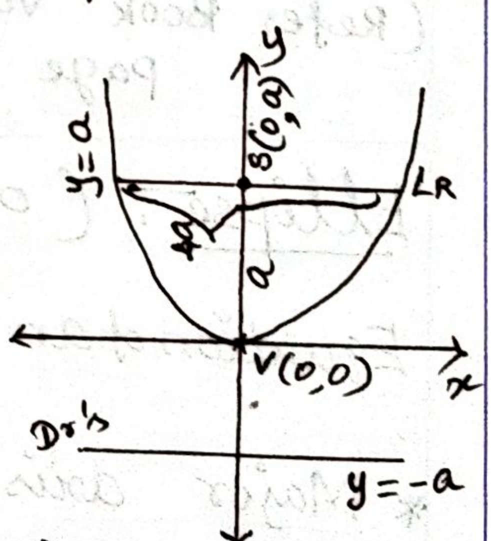
iv) Latus Rectum Eqn (LRE) : $x=-a$

v) Directrix (Dr's) : $x=a$

vi) Latus Rectum Length (LRL) : $4a$

3) Parabola open the Upward:

$$x^2 = 4ay$$



i) Axis : $x=0$

ii) vertex : $V(0,0)$

iii) Focus : $S(0,a)$

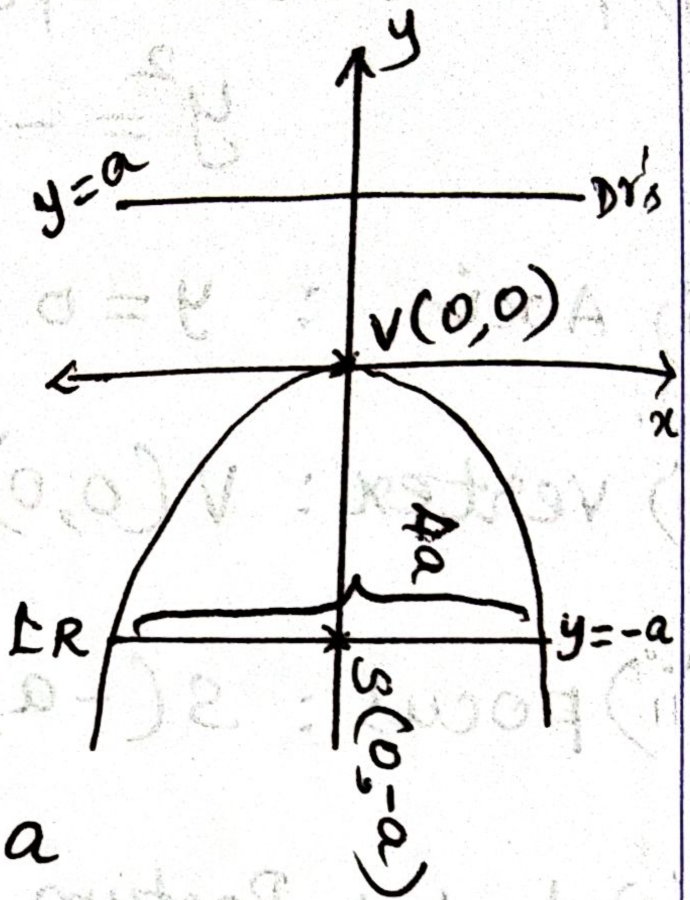
iv) Latus Rectum Eqn (LRE) : $y=a$

v) Directrix (Dr's) : $y=-a$

vi) Latus Rectum Length (LRL) : $4a$

4) Parabola open the Downward:

$$x^2 = -4ay$$



i) Axis : $x = 0$

ii) Vertex : $V(0,0)$

iii) Focus : $S(0,-a)$

iv) Latus Rectum Eqn (LRE) : $y = -a$

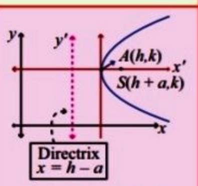
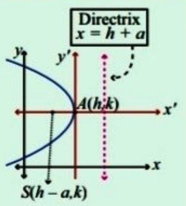
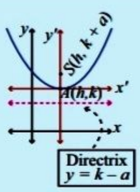
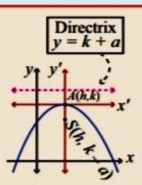
v) Directrix (Dr's) : $y = a$

vi) Latus Rectum Length (LRL) : $4a$

(ii) Parabolas with vertex at (h,k)

When the vertex is (h,k) and the axis of symmetry is parallel to x-axis, the equation of the parabola is either $(y-k)^2 = 4a(x-h)$ or $(y-k)^2 = -4a(x-h)$ (Fig. 5.19, 5.20).

When the vertex is (h,k) and the axis of symmetry is parallel to y-axis, the equation of the parabola is either $(x-h)^2 = 4a(y-k)$ or $(x-h)^2 = -4a(y-k)$ (Fig. 5.21, 5.22).

Equation	Graph	Vertices	Focus	Axis of symmetry	Equation of directrix	Length of latus rectum
$(y-k)^2 = 4a(x-h)$	 (a) The graph of $(y-k)^2 = 4a(x-h)$ Fig. 5.19	(h,k)	$(h+a,0+k)$	$y=k$	$x=h-a$	$4a$
$(y-k)^2 = -4a(x-h)$	 (b) The graph of $(y-k)^2 = -4a(x-h)$ Fig. 5.20	(h,k)	$(h-a,0+k)$	$y=k$	$x=h+a$	$4a$
$(x-h)^2 = 4a(y-k)$	 (c) The graph of $(x-h)^2 = 4a(y-k)$ Fig. 5.21	(h,k)	$(0+h,a+k)$	$x=h$	$y=k-a$	$4a$
$(x-h)^2 = -4a(y-k)$	 (d) The graph of $(x-h)^2 = -4a(y-k)$ Fig. 5.22	(h,k)	$(0+h,-a+k)$	$x=h$	$y=k+a$	$4a$

Ellipse : $(0<e<1)$

Equation of an ellipse in standard form :

* **Major axis parallel to the x-axis**

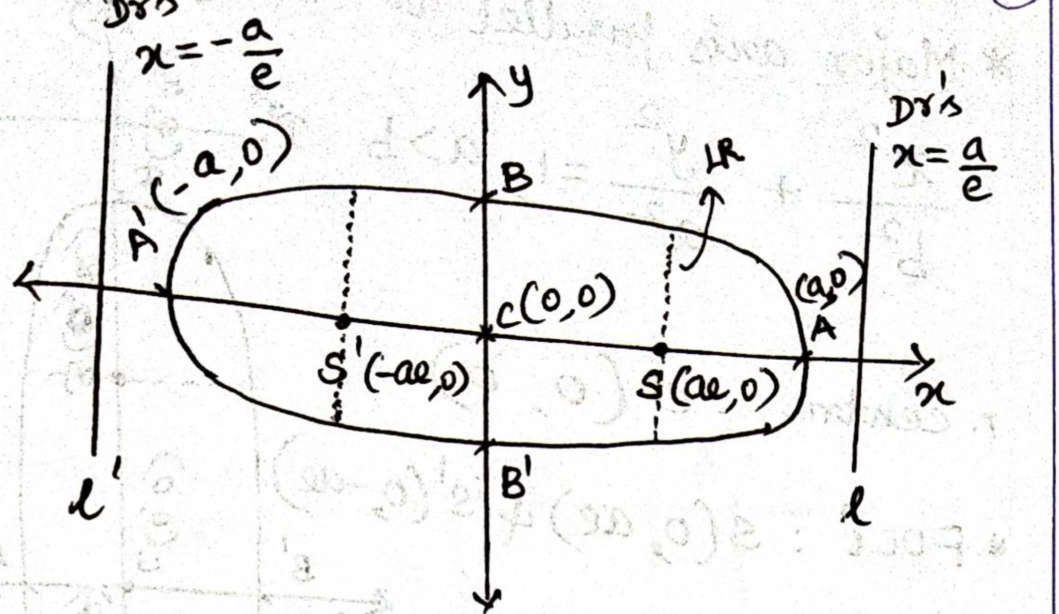
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a>b,$$

$c = ae$, $b^2 = a^2(1-e^2)$,

$c^2 = a^2 - b^2$

$$\frac{a}{e} = \frac{a^2}{c}$$

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$



⇒ The length of major axis $AA' = 2a$

⇒ The length of minor axis $BB' = 2b$

⇒ The semi major axis $CA = CA' = a$ and
$CB = b = CB'$

⇒ The End points of latus rectum L and L' are $\left(ae, \frac{b^2}{a}\right)$ & $\left(ae, -\frac{b^2}{a}\right)$

⇒ The Length of Latus Rectum $LL' = \frac{2b^2}{a}$

* Centre : $C(0,0)$
* Foci : $S(ae,0)$, $S'(-ae,0)$
* Vertices : $A(a,0)$, $A'(-a,0)$
* Directrix : $x = \pm a/e$
* Focus distance : $SS' = 2ae$

*** Major axis parallel to the y-axis :**

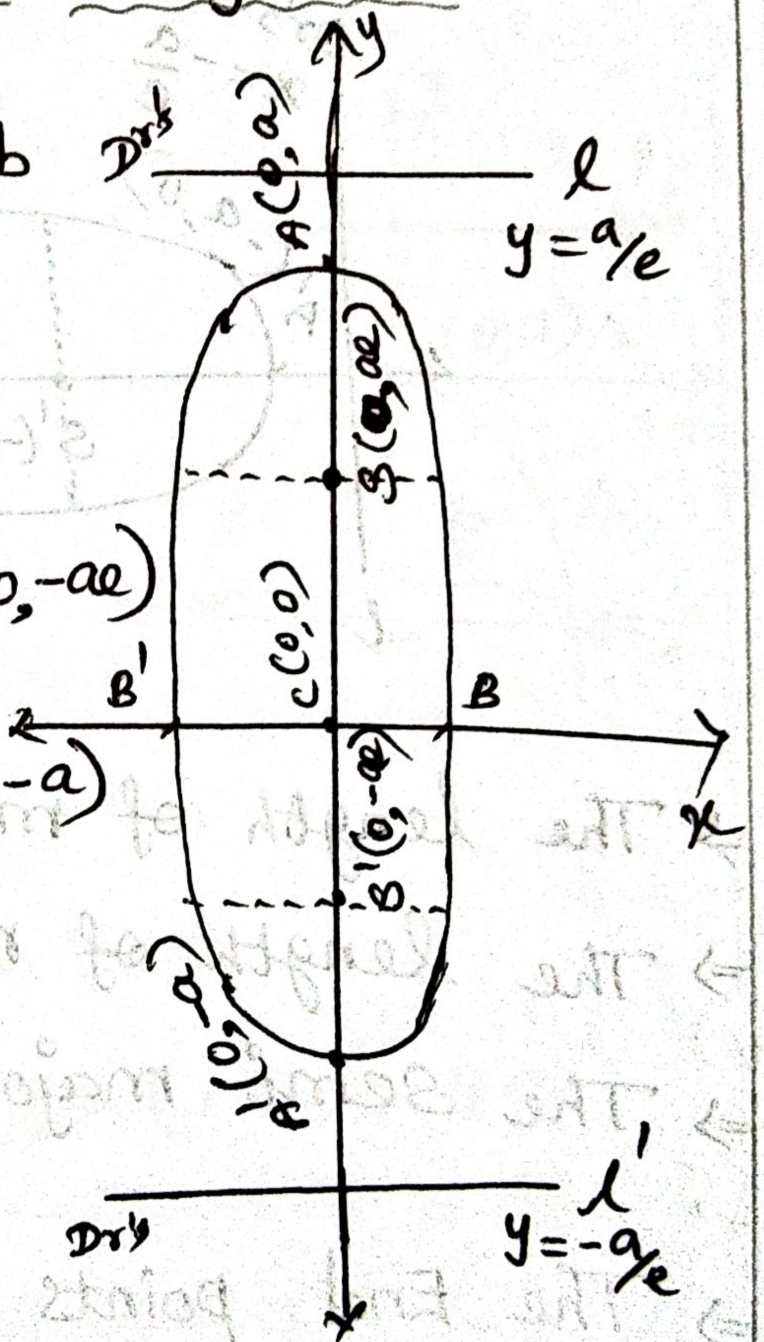
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad a > b$$

* centre : $c(0, 0)$

* Foci : $S(0, ae)$ & $S'(0, -ae)$

* vertices : $A(0, a)$ & $A'(0, -a)$

* Directrix : $y = \pm \frac{a}{e}$



(ii) Types of ellipses with centre at (h,k)

(a) Major axis parallel to the x-axis

From Fig. 5.24

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a>b$$

The length of the major axis is $2a$. The length of the minor axis is $2b$. The coordinates of the vertices are $(h+a,k)$ and $(h-a,k)$, and the coordinates of the foci are $(h+c,k)$ and $(h-c,k)$ where $c^2=a^2-b^2$.

(b) Major axis parallel to the y-axis

From Fig. 5.25

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1,\ a>b$$

The length of the major axis is $2a$. The length of the minor axis is $2b$. The coordinates of the vertices are $(h,k+a)$ and $(h,k-a)$, and the coordinates of the foci are $(h,k+c)$ and $(h,k-c)$, where $c^2=a^2-b^2$.

<table>
<tr><th>Equation</th><th>Centre</th><th>Major Axis</th><th>Vertices</th><th>Foci</th></tr>
<tr><td>

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $\boxed{a^2>b^2}$

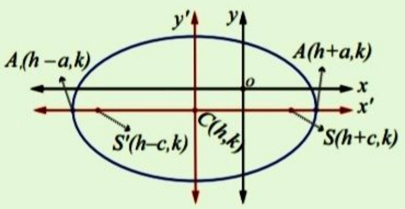


Fig.5.24

(a) Major axis parallel to the x-axis Foci are c units right and c units left of centre, where $c^2=a^2-b^2$.

</td><td>(h,k)</td><td>parallel to the x-axis</td><td>$(h-a,k)$
$(h+a,k)$</td><td>$(h-c,k)$
$(h+c,k)$</td></tr>
<tr><td>

$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ $\boxed{a^2>b^2}$

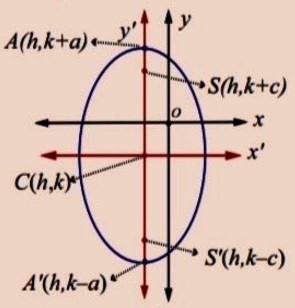


Fig.5.25

(b) Major axis parallel to the y-axis Foci are c units right and c units left of centre, where $c^2=a^2-b^2$.

</td><td>(h,k)</td><td>parallel to the y-axis</td><td>$(h,k-a)$
$(h,k+a)$</td><td>$(h,k-c)$
$(h,k+c)$</td></tr>
</table>

Hyperbola : $(e>1)$

standard equation of the hyperbola with centre $(0,0)$:

a) Transverse axis parallel to the x-axis :

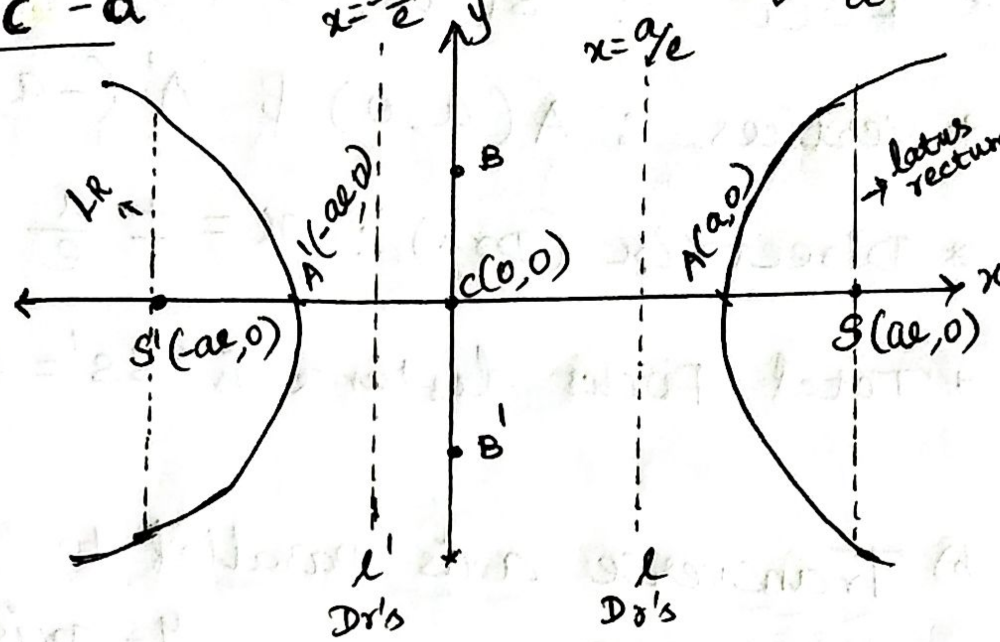
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$c = ae, \quad b^2 = a^2(e^2-1)$ | $\frac{a}{e} = \frac{a^2}{c}$

$c^2 = a^2 + b^2.$

$e = \sqrt{\frac{a^2+b^2}{a^2}}$

$\underline{b^2 = c^2 - a^2}$



⇒ The line Segment AA' is the transverse axis of length $2a$
(i.e) $AA' = 2a$

⇒ The line segment BB' is the Conjugate axis of length $2b$
(i.e) $BB' = 2b$

⇒ The line segment CA = The line segment CA' = semi transverse axis = a and the line segment CB = the line segment CB' = semi conjugate axis = b
(i.e) $CA = CA' = a$ & $CB = CB' = b$

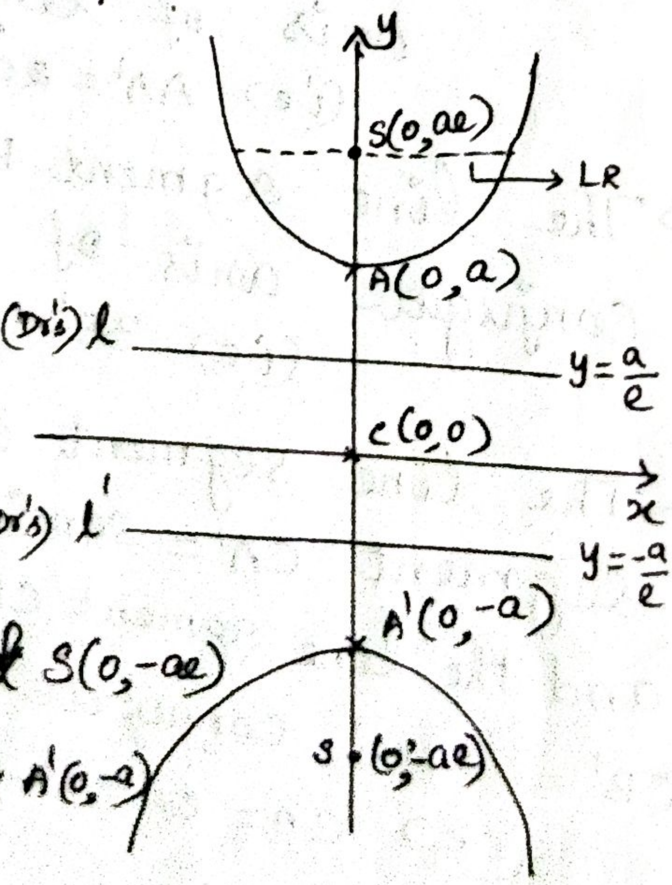
⇒ The length of Latus Rectum $LL' = \frac{2b^2}{a}$

⇒ Distance of Foci $SS' = 2ae = 2c$

* Centre : $C(0,0)$
* Foci : $S(ae,0)$ & $S'(-ae,0)$
* Vertices : $A(a,0)$ & $A'(-a,0)$
* Directrix (Dr's) : $x = \pm \frac{a}{e}$
* Total Focus distance is $SS' = 2ae$.

b) Transverse axis parallel to the y- axis :

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



* Centre $c:(0,0)$
* Foci : $S(0,ae)$ & $S(0,-ae)$
* Vertices : $A(0,a)$ & $A'(0,-a)$
* Directrix (Dr's) : $y = \pm \frac{a}{e}$

(ii) Types of Hyperbola with centre at (h, k)

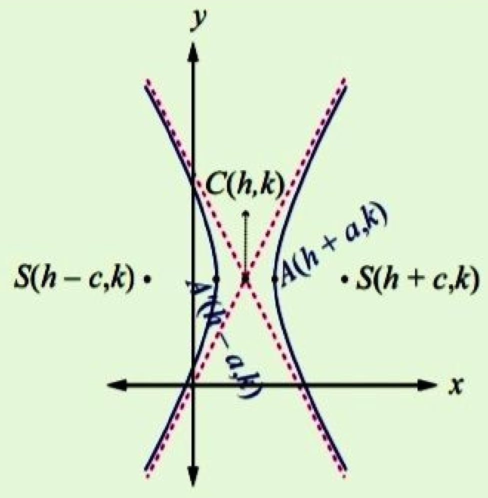


Fig. 5.29
(a) transverse axis parallel to the x-axis

(a) Transverse axis parallel to the x-axis.

The equation of a hyperbola with centre C (h,k) and transverse axis parallel to the x-axis (Fig. 5.29) is given by $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$.

The coordinates of the vertices are $A(h+a,k)$ and $A'(h-a,k)$. The coordinates of the foci are $S(h+c,k)$ and $S'(h-c,k)$ where $c^2=a^2+b^2$.

The equations of directrices are $x=\pm\frac{a}{e}$.

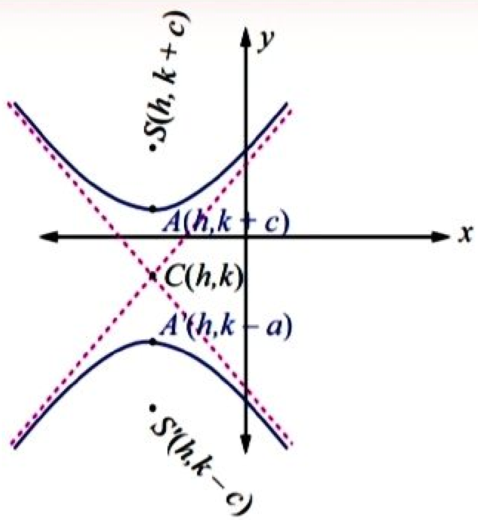


Fig. 5.30
(b) transverse axis parallel to the y-axis

(b) Transverse axis parallel to the y-axis

The equation of a hyperbola with centre $C(h,k)$ and transverse axis parallel to the y-axis (Fig. 5.0) is given by

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1.$$

The coordinates of the vertices are $A(h,k+a)$ and $A'(h,k-a)$. The coordinates of the foci are $S(h,k+c)$ and $S'(h,k-c)$, where $c^2=a^2+b^2$.

The equations of directrices are $y=\pm\frac{a}{e}$.

Remark